**On Ribet's isogeny for Jacobians of modular curves of small levels**

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Let $D$ be the reduced discriminant of an indefinite quaternion algebra over $\mathbb{Q}$, and let $X_D/\mathbb{Q}$ be the associated Shimura curve. Let $X_0(D)$ be the classical modular curve of level $D$. Ribet proved that there is an isogeny

$$\mathrm{Jac}(X_0(D))^{\mathrm{new}} \to \mathrm{Jac}(X_D)$$

defined over $\mathbb{Q}$, by proving that these abelian varieties have Galois isomorphic Tate modules. Unfortunately, the proof provides no information about the isogenies beyond their existence. In the case when $D$ is a product of two primes, Ogg conjectured that there is a Ribet isogeny whose kernel is an explicit subgroup of the cuspidal divisor group of $\mathrm{Jac}(X_0(D))$.

This conjecture remains largely open, except in the cases when $\dim(\mathrm{Jac}(X_D)) \leq 3$. In these latter cases, González and Rotger, and González and Molina verified Ogg's conjecture by explicitly computing the periods of the abelian varieties in question, which itself relies on the explicit computation of the equations of the corresponding Shimura curves $X_D$ over $\mathbb{Q}$. (The fact that $X_D$ is hyperelliptic when its genus is $\leq 3$ is used here.)

In a joint work with Fu-Tsun Wei, we verify Ogg conjecture in some cases when $\dim(\mathrm{Jac}(X_D)) > 3$ and $X_D$ is not hyperelliptic. Our approach is very different from that of González, Molina and Rotger; it crucially relies on the study of the Hecke algebra acting on $J_0(D)$ and requires very little information about the Shimura curve.